Beam Design - LEDGER ANALYSIS

1. Beam Data

Load Type: Uniform Dist. Load  
Support: Simple Beam  
Beam Type: Sawn Lumber  
Species: Douglas Fir-Larch  
Grade: DF No.2  
Size: 4 x 10  
Design Span (L): 27.75 ft.  
Clear Span: 27.50 ft.  
Total Span: 28.00 ft.  
Bearing (l): 3 in.  
Quantity (N): 1

2. Design Loads

Live Load: 35 plf  
Dead Load: 5 plf  
Selfweight: 213.4 lbs  
Dist. Selfweight: 7.69 plf  
Total Weight: 215.3 lbs

3. Design Options

Lateral Support: braced  
Defl. Limits: 360|240  
Load Duration: 1.15  
Exposure: dry  
Temperature: T <= 100°F  
Orientation: Vertical  
Incised Lumber: No  
Rep. Members: No

4. Design Assumptions and Notes

Bending Stress: Parallel to Grain  
Notes: LEDGER ANALYSIS

5. Adjustment Factors

<table>
<thead>
<tr>
<th>Factor</th>
<th>Description</th>
<th>Fb</th>
<th>Ft</th>
<th>Fv</th>
<th>Fc</th>
<th>Fc⊥</th>
<th>E/Emin</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD</td>
<td>Load Duration Factor</td>
<td>1.15</td>
<td>1.15</td>
<td>1.15</td>
<td>1.15</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CM</td>
<td>Wet Service Factor</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<tr>
<td>Ct</td>
<td>Temperature Factor</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>CL</td>
<td>Beam Stability Factor</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CF</td>
<td>Size Factor</td>
<td>1.2</td>
<td>1.1</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cfu</td>
<td>Flat Use Factor</td>
<td>1.1d</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ci</td>
<td>Incising Factor</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Cr</td>
<td>Repetitive Member Factor</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

a) Adjustment factors per AWC NDS 2015 and NDS 2015 Supplement.  
b) When (Fb)(Ft) ≤ 1,150 psi, CM = 1.0.  
c) When (Ft)(Fc) ≤ 750 psi, CM = 1.0.  
d) Only applies when sawn lumber or glulam beams are loaded in bending about the y-y axis.
6. Beam Calculations

Determine reference design values, sectional properties and self weight of beam:

\[ A = b \times d \]

where:

- \( b \) = Breadth of rectangular beam in bending (in.)
- \( d \) = Depth of rectangular beam in bending (in.)
- \( A \) = Cross sectional area of beam \((\text{in.}^2)\)
- \( S_x \) = Section modulus about the X-X axis \((\text{in.}^3)\)
- \( S_y \) = Section modulus about the Y-Y axis \((\text{in.}^3)\)
- \( I_x \) = Moment of inertia about the X-X axis \((\text{in.}^4)\)
- \( I_y \) = Moment of inertia about the Y-Y axis \((\text{in.}^4)\)

\[
\begin{align*}
  b &= 3.500 \text{ in.} \\
  d &= 9.250 \text{ in.} \\
  A &= 3.500 \times 9.250 = 32.38 \text{ in.}^2 \\
  S_x &= \frac{(3.500)(9.250)^2}{6} = 49.91 \text{ in.}^3 \\
  S_y &= \frac{(3.500)^2(9.250)}{6} = 18.89 \text{ in.}^3 \\
  I_x &= \frac{(3.500)(9.250)^3}{12} = 230.84 \text{ in.}^4 \\
  I_y &= \frac{(3.500)^3(9.250)}{12} = 33.05 \text{ in.}^4
\end{align*}
\]

Reference Design Values from Table 4A NDS Supplement (Reference Design Values for Visually Graded Dimension Lumber, 2" - 4" thick).

<table>
<thead>
<tr>
<th>Species &amp; Grade</th>
<th>( F_b )</th>
<th>( F_t )</th>
<th>( F_v )</th>
<th>( F_{c,\perp} )</th>
<th>( F_c )</th>
<th>( E )</th>
<th>( E_{\text{min}} )</th>
<th>( G )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DF No.2</td>
<td>900</td>
<td>575</td>
<td>180</td>
<td>625</td>
<td>1350</td>
<td>1600000</td>
<td>5800000</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The following formula shall be used to determine the density of wood \((\text{lbs/ft}^3)\). \((\text{NDS Supplement Sec. 3.1.3})\)

\[ \rho_w = \text{Density of wood (lbs/ft}^3\) \]

where:

- \( \rho_w \) = Density of wood \((\text{lbs/ft}^3)\)
- \( G \) = Specific gravity of wood (dimensionless)
- m.c. = Moisture content of wood (percentile)

\[ G = 0.5 \]

m.c. = 19 %  (Max. moisture content at dry service conditions)
**Volume**

\[ \text{Volume}_{\text{total}} = N[A \times (L + l_b)] = 1 \times [32.38 \times (333.00 + 3)] \times (12 \text{ in./ft.})^3 = 6.30 \text{ ft}^3 \]

\[ \text{Volume}_{\text{span}} = N[A \times L] = 1 \times [32.38 \times 333.00] \times (12 \text{ in./ft.})^3 = 6.24 \text{ ft}^3 \]

**Total Weight (W_T)**

\[ W_T = \rho_w \times \text{Volume}_{\text{total}} = 34.20 \times 6.30 = 215.3 \text{ lbs} \]

**Self Weight (W_S)**

\[ W_S = \rho_w \times \text{Volume}_{\text{span}} = 34.20 \times 6.24 = 213.4 \text{ lbs} \]

**Distributed Self Weight (w_s) = 7.69 plf**

**Load, Shear and Moment Diagrams:**

![Load, Shear and Moment Diagrams](image-url)
1.) **Bending:**

Members subject to bending stresses shall be proportioned so that the actual bending stress or moment shall not exceed the adjusted bending design value:

\[ f_b \leq F_{b}' \quad (NDS \text{ Sec. 3.3.1}) \]

where:

\[ f_b = \frac{M}{S} \]
\[ F_{b}' = F_b(C_D)(C_M)(C_L)(C_F)(C_i)(C_r) \]

Beam is braced laterally along its compression edge. Lateral stability is not a consideration:

\[ C_L = \text{Beam Stability Factor} = 1.0 \]
\[ F_{bx}' = (900)(1.15)(1)(1)(1.2)(1)(1) = 1242.0 \text{ psi} \]
\[ f_b = = 1103.7 \text{ psi} \]
\[ f_b = 1103.7 \text{ psi} < F_{bx}' = 1242.0 \text{ psi} \quad (CSI = 0.89) \quad \text{OK} \]
2.) Shear:

Members subject to shear stresses shall be proportioned so that the actual shear stress parallel to grain or shear force at any cross section of the bending member shall not exceed the adjusted shear design value:

\[ f_v \leq F_v' \quad (NDS \text{ Sec. 3.4.1}) \]

where:

\[ f_v = \]

\[ F_v' = F_v(C_D)(C_M)(C_l)(C_i) \]

\[ F_{vx}' = (180)(1.15)(1)(1)(1) = 207.00 \text{ psi} \]

Shear Reduction: For beams supported by full bearing on one surface and loads applied to the opposite surface, uniformly distributed loads within a distance, \( d \), from supports equal to the depth of the bending member shall be permitted to be ignored. For beams supported by full bearing on one surface and loads applied to the opposite surface, concentrated loads within a distance equal to the depth of the bending member from supports shall be permitted to be multiplied by \( x/d \) where \( x \) is the distance from the beam support face to the load. See NDS 2015, Figure 3C.

\[ f_v* = 28.95 \text{ psi} \]

\[ f_v* = 28.95 \text{ psi} < F_{vx}' = 207.00 \text{ psi} \quad (CSI = 0.14) \quad \text{OK} \]

No Reduction in Shear (conservative):

\[ f_v = 30.66 \text{ psi} \]

\[ f_v = 30.66 \text{ psi} < F_{vx}' = 207.00 \text{ psi} \quad (CSI = 0.15) \quad \text{OK} \]

3.) Deflection:

Bending deflections calculated per standard method of engineering mechanics for live load and total load:

LL Allowable: \( L/360 \)

TL Allowable: \( L/240 \)

\[ E_x' = E_x(C_M)(C_l)(C_i) = 1600000(1)(1)(1) = 1600000 \text{ psi} \]
ΔLL = 1.26 in.

(L/d)LL = 333.00 / 1.26 = 263

ΔLL = 1.26 in = L/263 > L/360  ? NG

ΔTL = 1.72 in.

(L/d)TL = 333.00 / 1.72 = 193

ΔTL = 1.72 in = L/193 > L/240  ? NG

4.) Bearing:

Members subject to bearing stresses perpendicular to the grain shall be proportioned so that the actual compressive stress perpendicular to grain shall be based on the net bearing area and shall not exceed the adjusted compression design value perpendicular to grain:

\[ f_{c\perp} \leq F_{c\perp}' \quad (NDS \text{ Sec. } 3.10.2) \]

where:

\[ f_{c\perp} = \]

\[ F_{c\perp}' = F_{c\perp}(C_M)(C_I) \]

\[ F_{c\perp}x' = (625)(1)(1)(1) = 625.00 \text{ psi} \]

\[ A_b = b \times l_b = 3.5 \times 3 = 10.50 \text{ in}^2 \]

\[ f_{c\perp} = 63.6 \text{ psi} \]

\[ f_{c\perp} = 63.6 \text{ psi} < F_{c\perp}x' = 625.00 \text{ psi} \quad (\text{CSI} = 0.10) \quad ? \text{OK} \]