Design of Metal Plate Connected Wood Truss Joints for Moment

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Summary

The Truss Plate Institute (TPI) published a methodology for designing metal plate connected wood truss joints for moment in 2002. Subsequent use and further research showed the need for modification of this method to recognize the interaction between axial compression and moment stresses and to recognize the effect of plates located off-center. This article reports on further development of this methodology, including consideration of wood compression capacity due to direct contact, consideration of steel yielding and plastic behavior in tension, consideration of steel buckling capacity in compression, correlation of the predictive equations with test data, and further modification for design application. These equations permit reasonably accurate establishment of allowable moment capacities across the full range of axial loadings of truss joints, from full allowable compression loading to full allowable tension loading.

1. Introduction

Design equations for moment capacity of the steel cross-section of metal plate connected wood truss joints were recently established as shown below (TPI, 2002).

The moment applied to a metal connector plate used in chord splices shall not exceed the moment capacity defined as follows:

\[
M_a = 0.45t_1R_tF_u(0.5W_p + y_c)^2 + 0.15d_2C(0.5d_1 - y_c)^2 - P_ty_c
\]

where:

\[y_c = \left[\frac{(0.25d_1d_2C) + 2.22P_t - (t_1W_pR_tF_u)}{[2(t_1R_tF_u) + (0.5d_2C)]}\right]
\]

\[M_a = \text{maximum allowable moment in the plane parallel to the metal connector plate surface (lb-in). When } M_a < 0 \text{ per E8.8-1, } M_a \text{ shall be considered to equal zero.}
\]

\[R_t = \text{plate tensile effectiveness ratio for load in a direction perpendicular to joint}
\]
\[ F_u = \text{plate steel ultimate tensile strength (psi)} \]
\[ W_p = \text{plate dimension parallel to joint line, not to exceed } d_1 \]
\[ y_c = \text{distance to neutral axis with respect to chord centerline (not to exceed +/- 0.5d_1).} \]
\[ C = 1.2F_c^* \]
\[ P_t = \text{tension force normal to joint line, positive for tension (zero if compression).} \]

(Also, TPI 1 defines: \( t_1 = \text{plate design thickness}; d_1, d_2 = \text{wood cross-section dimensions in, and perpendicular to, plane of truss, respectively.} \) TPI 1 Commentary clarifies \( C = 1.2F_c^* \) was based on chord splices cut perpendicular to grain where wood-to-wood butting force due to moment is aligned parallel to grain. Application to other angles should base \( C \) on a Hankinson interpolation between \( 1.2F_c^* \) and \( F_{cperp} \).)

The above equation was based upon prior work by O’Regan et al. (1998) that built on the work of Noguchi (1980). Two shortcomings identified in the above equation were as follows.

1) It did not consider the effect of compression force on the moment capacity of the truss plate steel section. This lack of consideration of compression was addressed by ignoring compression, which was believed to be conservative as the ultimate failure mode of tested joints has always been steel tensile fracture, so application of compression can safely be ignored. This has been deemed to be too conservative with respect to its effect on joints that have been known to work by long-standing use, and by preliminary testing by a TPI member firm demonstrated 50 percent increases in moment capacity for one joint configuration with the application of moderate compressive force.

2) It did not recognize that plates may be positioned elsewhere than centered on the lumber centerline. This was due to all designers specifying chord splices on chord centerlines. However, TPI (TPI, 2002) also requires truss designers to identify positioning tolerances as limited by design equations, but the moment equation does not consider other positions.

A Task Group established by TPI improved the current equation to resolve these and other shortcomings. This article describes the structural theory, confirming test data, and adjustments necessary for design application developed by this Task Group.

2. Methods

2.1 Theory Development

Similar work to that done previously (O’Regan et al, 1998) was performed but modifying the assumptions made regarding wood and steel stress distributions, plate positioning, and the location of the neutral axis, as follows.

- The plate position considered locations other than at the centerline of the lumber. Prior work considered only a centered plate.
- The neutral axis location consideration including positions within the truss plate, as well as outside of the truss plate. Prior work considered only a location within the truss plate.
- Changes in the extent or shape of the wood and steel stress distributions. Prior work considered the steel acting only in tension and the wood acting only in compression, with both materials attaining their limiting stress simultaneously, and assuming one of three different stress distribution models: linearly varying (elastic) stress distribution in the wood and in the tension portion of the plate, a linearly varying stress distribution in the wood in compression with a constant (plastic) stress distribution in the tension portion of the plate, and a constant stress distribution in both the wood in compression and in the steel in tension. Changes considered are described further below.
The assumptions of linear elastic and constant plastic stress distributions seem reasonable even while considering axial compression load, but it was considered necessary to try other models in an effort to best match test data. This included the following variations.

- Consideration of plate compression capacity. Past testing by Kirk et al (1989) and data from TPI members was thought to substantiate plate compression capacity. This data showed that compression buckling strength can equal or exceed net section tensile yield capacity. There was also some data substantiating that post-buckling strength of plates was about 40 to 50 percent of buckling capacity. Considering this compression capacity in moment design theory was thought likely to improve the consistency of predicted capacity with the tested capacity.

- Consideration of the strain-hardening region of the steel tensile stress distribution. Previous work considered a constant-stress rectangular stress block for the steel in tension for the plastic distribution. It was thought that a trapezoidal distribution extending from yield stress ($F_y$) to ultimate stress ($F_u$) may result in better compatibility with test data.

- Enforce strain compatibility. It may be inappropriate to presume that both wood and steel reach their limiting stress simultaneously. This is especially true for elastic stress models.

- Consider plastic compression stress distributions with elastic tensile stress distributions. This may be especially appropriate for joints governed by compression strength as such joints may never cause a plastic state of stress in the tension portion of the plate. While ultimate moment failure is typically a result of tensile fracture of the steel, joints primarily under compression are likely to be governed by a wood compression strength limit state.

This resulted in the development of 22 distinct equations based on neutral axis locations and stress distributions outlined in Table 1. As Case 4 from Table 1 was recommended for further use, its derivation follows. Figure 1 shows joint geometry and stress distributions for this case.

![Figure 1. Joint geometry & assumed stress distribution for Case 4.](image)

Variable labels identical to those from TPI (2002) were used, except that the neutral axis location was defined relative to the compression edge of the wood (now labeled “y”) and a new variable, labeled “z”, was established to account for plate position and set equal to the dimension from the compression edge of the wood to the compression edge of the plate. The force normal to the joint, $P$, is now permitted to be negative when compression force exists.

The wood and steel stresses in compression were modeled with constant distributions equal to the maximum crushing strength of the wood, $F_{mcs}$, and a fraction of the steel tensile yield strength, $a*F_y$. The value of “a” was set to 0.4 based on the discussion earlier. The steel tensile
stress was modeled with a trapezoidal distribution extending from yield (F_y) at the n.a. to ultimate stress (F_u) at the extreme tensile edge of the plate. These distributions are simplified from the true stress-strain curves of steel and wood, but are reasonably accurate. The steel stress distribution was separated into two stress blocks for ease of computation: a rectangular block (distribution 1) with a constant stress, F_y, and a triangular stress block (distribution 2) extending from zero at the n.a. to a maximum of (F_u – F_y). The net steel tension forces of distributions 1 and 2, T1 and T2, respectively, and the net compression forces on the steel and wood, Cs and Cw, respectively, can be calculated as the relevant cross-sectional area times the relevant average stress, as follows, where R_t is the tensile efficiency of the plate in a direction perpendicular to the joint line and adjusts for the presence of punched slots.

\[
\begin{align*}
\text{Force} & = \text{Area} \ast \text{Average Stress} \\
T1 & = 2t_1R_t(W_p-y+z) \ast F_y \\
T2 & = 2t_1R_t(W_p-y+z) \ast \frac{(F_u-F_y)}{2} \\
Cs & = 2t_1R_t(y-z) \ast aF_y \\
Cw & = d_2y \ast F_{mcs}
\end{align*}
\]

The neutral axis location at failure can be determined by force equilibrium from:
\[
\Sigma F = 0 = T1 + T2 - Cs - Cw - P
\]
Substituting in the terms above and solving for y provides:
\[
y = \frac{t_1R_t[F_y(W_p+z) + F_y(W_p+z(1+2a))] - P}{t_1R_t[(F_y(1+2a)+F_u)] + d_2F_{mcs}}
\]
The ultimate moment capacity, $M_u$, can then be determined by moment equilibrium. The following equation results from summing moments about the centerline of the lumber, using moment arms as the distance between the wood centerline and the centroid of each stress distribution (center of distribution for rectangular stress blocks, or 2/3-point for triangular stress blocks). These moment arms are as follows.

Force T1:

$$\frac{(W_p - (y - z))/2 - (d_1/2 - y)}{2} = \frac{(W_p + y + z - d_1)/2}{2}$$

Force T2:

$$\frac{2(W_p - (y - z))/3 - (d_1/2 - y)}{6} = \frac{(4W_p + 2y + 4z - 3d_1)/6}{6}$$

Force Cs:

$$\frac{(y - z)/2 + (d_1/2 - y)}{2} = \frac{(d_1 - y - z)/2}{2}$$

Force Cw:

$$\frac{y/2 + (d_1/2 - y)}{2} = \frac{(d_1 - y)/2}{2}$$

Summing the above moment constituents (forces times their respective distances above) gives a total moment equal to:

$$M = \frac{T_1(W_p+y+z-d_1)}{2} + \frac{T_2(4W_p+2y+4z-3d_1)}{6} + \frac{Cs(d_1-y-z)}{2} + \frac{Cw(d_1-y)}{2}$$

Other equations for other cases were developed in the same manner. For elastic stress distributions, strain compatibility was enforced.

### 2.2 Joint Testing

Test data (O’Regan et al, 1998) existed for joints subjected to moment and tension, but testing was necessary to generate data for joints subject to moment and compression. Three task group members with available testing equipment performed such testing using splice joints shown in Figures 2 and 3 with parameters shown in Table 2. It was agreed that this represented a reasonable range of truss plate types, steel properties, wood and plate sizes, and ratios of moment:axial stresses in order to confirm that the theory could be applied across the range of parameters that occur in design practice.

Each compression joint was built of 14 inch long members at the splice, with no gap, resulting in a total joint length of 28 inches, with short members (6 to 7 inch long 2x4 or wider members) plated to the ends as needed to allow for eccentric axial loading. These short lengths minimized the secondary moment occurring due to bending deflection (P-delta effect), but bending deflection measurements at the joint line were also made in order to explicitly account for this effect. Pure bending joints were suggested to
Table 2. Variables for combined compression & moment tests (factorial design)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood Size:</td>
<td>2 per lab: 2x4 and either 2x6, 2x8 or 2x10, each 14 in long.</td>
</tr>
<tr>
<td>Wood Species/Grade:</td>
<td>1/size, selected by lab (Lab A: SP 2x4/DF-L 2x10; Lab B: SP 2x4 &amp; 2x6; Lab C: SP or TJ LSL 2x4 / SP 2x8)</td>
</tr>
<tr>
<td>Plate Type:</td>
<td>2: standard grade (Fy/Fu = 40/55 ksi) and high grade (Fy/Fu = 60/70 ksi) 20 gage truss plates. All plates had slots in rows, except Lab A high grade and Lab C plates were fully staggered.</td>
</tr>
<tr>
<td>Plate Position:</td>
<td>2 = symmetrical lengthwise and offset 1 inch laterally to wood compression edge.</td>
</tr>
<tr>
<td>Moment:Axial Ratio:</td>
<td>4 = Pure bending tests, as well as eccentric compression tests where compression load is applied at three separate eccentricities such as 1.75, 3.5 and 7 inches</td>
</tr>
<tr>
<td>Replicates:</td>
<td>3</td>
</tr>
<tr>
<td>Total Test Joints</td>
<td>$2 \times 1 \times 2 \times 2 \times 4 \times 3 = 96$ joints per lab</td>
</tr>
</tbody>
</table>

2.3 Results and Modification of Equation for Design Purposes

The equation from Case 4 compared well to previous data from O’Regan et al (1998) as well as the new data. The predictive equation was initially scaled by dividing by 2.82 so as to give a lowest ratio of any test set average capacity to the predicted capacity (using the divisor) of 2.0. These ratios, prior to scaling, averaged 1.0 indicating good agreement on average between the equation and tested performance, although the values for each set ranged from 0.71 to over 3. The difference in the divisor between scaling based on the lowest ratio of any joint compared to the average ratio for each set was found to be only six percent. This six percent variation is small compared to typical cold-formed steel industry practice per AISI (American Iron and Steel Institute, 2001), which permits design capacities from testing based on application of a divisor to the mean and recognizes that individual values may vary by more than 15 percent. It is of interest to compare the selected divisor of 2.5 with the divisor that would result from this same COV per the AISI standard for cold-formed steel members, which was found to be 2.13 for the 0.214 coefficient of variation (COV) in this data (normalized by division by predicted capacity).
Based on these comparisons to AISI practice, the divisor of 2.82 was lowered to 2.5. This remains conservative relative to other cold-formed steel applications, but is appropriate given the target safety factor of 2.0. In addition to this divisor, the equation for neutral axis location was modified by scaling the applied force, \( P_t \), by 2. This was necessary as the force will be an allowable force, while the predictive equation is based on ultimate with a safety factor of 2.0.

The \( C \) parameter was also modified. The 1.2 factor in this term was modified to 1.7 in order to obtain a better estimate of average wood strength, instead of the fifth percentile estimate formerly used. This change was because of recognition that the predictive equation results in a divisor, or safety factor, of 2.0, far more than the 1.2 divisor prescribed by wood design standards for compression strength. The 1.7 factor was based upon the ratio of the mean value to the 5th percentile value for maximum crushing strength (MCS) of lumber, presuming normal distribution and an 18 percent COV on MCS (Forest Products Laboratory, 1999).

Figure 4 shows test data for the tested 2x4 joints along with curves representing the current TPI design equations and two versions of the new equation: one with the originally contemplated divisor of 2.82 and the original \( C \) value (1.2\( F_c^* \)) and one with the final divisor of 2.5 and the modified \( C \) value of 1.7\( F_c^* \). Only data from centered plates were used as the existing TPI 1-2002 equation is based on that plate positioning assumption. The plots show moment capacities determined across the full range of allowable axial capacities, meaning from axial compression forces as high as permitted based on allowable wood compression stresses to axial tension forces as high as permitted based on allowable plate steel stress.

**Fig. 4. Plots of Test Data and Capacities Predicted by Design Equations**
3 Further Discussion

The potential interaction of bending deflection in combination with axial load within a truss chord was considered. This causes a secondary moment, termed a P-delta interaction, and truss analysis and design typically accounts for this through the use of an amplification term on the moment in the design equations, rather than explicit analytical calculation. The same amplification used for wood design is suggested for plate design, with this amplification term linearly scaled down to zero as the truss plate joint moves from midpanel to the panel point. The Task Group also incorporated the effect of grain angle on the wood bearing strength into the C term, previously recognized only in commentary. These changes result in the recommended design equation below, using the terminology discussed earlier.

\[ M_a = C_{m} \left\{ T_1 (W_p+y+z-d_1) + T_2 (4W_p+2y+4z-3d_1) / 3 + C_s (d_1-z-y) + C_w (d_1-y) \right\} / 5 \]

where: 
\( C_{m} = \) For forces from analysis that produce moment without considering interaction of axial compression and transverse deflection on moment and when joint carrying moment is also subject to compression: \( 1 - (x/L)(f_c / F_{cEx}) \). For other situations: \( 1.0. \)
\( x = \) distance between splice and nearest panel point (in)
\( L = \) length of panel in which splice is located (in)
\( f_c, F_{cEx} = \) stresses per section 7.3.5.1 for wood members adjacent to splice (psi)
\( y = \) \( \{ t_1 R_t \{ F_{y} (1.8z+W_p ) + F_u (W_p+z) \} - 2P_t \} / \{ d_2 C + t_1 R_t (1.8F_{y}+F_u) \}, \) (in)
\( T_1 = 2t_1 R_t F_{y} (W_p-y+z) \)
\( T_2 = t_1 R_t (F_u-F_{y}) (W_p-y+z) \)
\( C_s = 0.8t_1 R_t F_{y} (y-z) \)
\( C_w = yd_2 C \)
\( C = F_{cperp} (1.7F_{c}^*) / (F_{cperp} \sin^2 \theta + 1.7F_{c}^* \cos^2 \theta) \) where \( F_{c}^* \) and \( F_{cperp} \) are allowable compr. stresses parallel and perp. to grain, and \( \theta \) is angle between joint and member (psi)

4 Conclusions

The new equation results in more accurate prediction of moment capacities, especially with the application of compression force and with plate positions off of the wood centerline.

5 References